Advances in Fuzzy Mathematics.
ISSN 0973-533XVolume 11, Number 2 (2016), pp. 113-127

http://www.ripublication.com

Rough lattice and Core in Rough data set Lattice

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Abstract

The chief objective of this study is to show the usefulness of Rough set theory in lattice theory. The aim of this paper is to introduce rough lattice, weighted rough lattice, rough data set lattice& weighted rough data set lattice. Also discuss about the reduct & core in rough data set lattice & weighted rough data set lattice.

Key words: Undefinable (rough) sub lattice, rough lattice, reduct and core, rough data set lattice, weighted rough data set lattice.

1. Introduction

Rough set theory, proposed by Zdzislaw Pawlak (1982) [6] & [7] models uncertainty by equivalence relations. The primary notion is the partitioning of the domain in to equivalence classes. Objects belonging to the same equivalence cannot be distinguished. Hence Rough set theory is a theory of multiple memberships.

A lattice is a convenient way of representing information involving relationship between objects. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a “Rough lattice”. This paper connects two research topics rough set and lattice theory, both of which have applications across a wide variety of field. Y. Jianfeng] [4] & L. Lan [3] have finding attribute reduction based on information entropy. This paper focus on the elimination of the redundant attributes in order to generate the effective lattice model and formulating the core of the lattice. The core is taken up the central structure of a rough data set lattice & weighted rough data set lattice..

2. Definition and main results

2.1 Definition –Rough set

A rough set is a formal approximation of a crisp set in terms of a pair of sets which the lower and upper approximation of the original set. Let U denote the set of objects called universe and let E be an equivalence relation on U. The pairs R = (U, E) is called an approximation space. For $u, v \in$U & $(u, v) \in$ E, u and v belong to the same equivalence class and we say that they are indistinguishable in R. The relation E is called an indiscernibility relation. Let $[x]_E$ denote an equivalence class of E containing element x, then lower & upper approximation for a subset X ⊆ U in A denoted by $\underline{R}$ (X) & $\overline{R}$ (X) respectively

Where

$\underline{R}$ (X) = $\{x \in U / [x]_E \subset X\}$

$\overline{R}$ (X) = $\{x \in U / [x]_E \cap X \neq \phi\}$

Thus if an object $x \in \underline{R}$ (X) then "x surely belongs to X in R"

If $x \in \overline{R}$ (X) then "x possibly belong to X in R"

2.2 Membership value:

The membership value of X is μ (X) = $\frac{|\underline{R}\ (X)|}{|\overline{R}\ (X)|}$

The membership value of each element of X in R $\mu_X\ (x) = \frac{|\ ([x]_E \cap X)|}{|\ [x]_E|}$

2.3 Results

1. If $\overline{R}$ (X) = $\underline{R}$ (X) then X is definable on attribute set R.
2. If $\overline{R}$ (X) $\neq$ $\underline{R}$ (X) then X is undefinable (roughly definable)on R (or)**Rough set** in R

a. If $\underline{R}$(X) $\neq \phi$ &$\overline{R}$ (X) $\neq$U then X is undefinable (R- roughly definable)on R

b. If $\underline{R}$ (X)) $\neq \phi$ &$\overline{R}$ (X) = U then X is internally undefinable (R_i) on R

c. If $\underline{R}$ (X) $= \phi$ &$\overline{R}$ (X) $\neq$U then X is externally undefinable (R_e) on R

d. If $\underline{R}$ (X) $= \phi$ &$\overline{R}$ (X) = U then X is totally undefinable (R_t) on R

2.4 Definition:

Let E be an equivalence relation U. Then E is called a congruence relation if $(x, y) \in E$ implies that $(x \vee a, y \vee a) \in$E and $(x \wedge a, y \wedge a) \in E$ for all $a \in$U.

2.5 Definition:

A poset $(P, \leq)$ is called a meet semi lattice if $\wedge\, b \in P\ \forall a, b \in P$. A poset $(P, \leq)$is called a join semi lattice if $a \vee b \in P\ \forall\ a, b \in P$

2.6 Definition:
Let A be a partially ordered set. A set X ⊆A is called a **down set** in A if $X = D\ (X)_A$ Where $D\ (X)_A = \{y \in A\ /y \leq x$ for some $x \in X\}$**.** A set X ⊆A is called a **up set** in A if $X = U\ (X)_A$ where $U\ (X)_A = \{y \in A/y \geq x$ for some $x \in X\}$

2.7 Rough Poset:
Let R = (U, E) be an approximation space. Let X be a subset of U. A rough poset (R (X), ≤) is a rough set together with a partial order relation ≤ on X. (ie) a relation which is reflexive, anti symmetric and transitive.

2.8 Meet semi rough lattice:
A rough poset (R (X), ≤) is called a meet semi rough lattice if $a \wedge b \in \overline{R}\ (X)\ \forall a, b \in X$

2.9 Join semi rough lattice:
A rough poset (R (X), ≤) is called a join semi rough lattice if $a \vee b \in \overline{R}$ (X) $\forall a, b \in$ X

2.10 Rough Lattice :
Let R = (U, E)be an approximation space. Let ∨ & ∧ be the operations defined on U. Let X be a subset of U. A rough poset R (X) is called a rough lattice. If the following properties are satisfied. $(i) a \wedge b \in \overline{R}\ (X)\ \forall a, b \in\ X\ (ii) a \vee b \in \overline{R}\ (X)\ \forall a, b \in\ X$

2.11 Weighted Rough Lattice :
Let R = (U, E)be an approximation space. Let E be an equivalence relation on U & A Rough lattice is a pair R (L) = ($\underline{R}$ (L), $\overline{R}$ (L)). A weighted rough lattice is of the form $R\ (L)_W$: (U, $\underline{R}, \overline{R}, \mu, \leq$) Where μ is a rough membership value
Define $\mu_1 : \underline{R} \rightarrow 1$, $\mu_2 : \overline{R} \rightarrow (0, 1]$, μ_3 : U- $\overline{R} \rightarrow 0$

2.12 Rough sub Lattice:
A non empty subset R (X) of rough lattice R (L) is called its rough sub lattice, if it is a rough lattice itself w. r. to the operations ∨ & ∧ (∧- lower bound, ∨-upper bound

2.13 Rough set Algebra [1]:
Let $\underline{R} : P\ (U) \rightarrow P\ (U)$& $\overline{R} : P\ (U) \rightarrow P\ (U)$ be the approximation operators then the system $(P\ (U), c, \underline{R}, \overline{R}, \cup, \cap)$ is called a rough set algebra.

3. Properties of rough lattices

3.1 Proposition: In any rough lattice (R (X), ≤) the following are true for any $a, b, c \in X$
$(L1)\ (a \wedge a) = a, a \vee a = a$
$(L2)(a \wedge b) = (b \wedge a), (a \vee b) = (b \vee a)$
$(L3)(\ a \wedge b) \wedge c = a \wedge (b \wedge c), (\ a \vee b) \vee c = a \vee (b \vee c)$

$(L4) a \wedge (a \vee b) = a, a \vee (a \wedge b) = a$

3.2 Proposition:

Let (U, E) be an approximation space. for every rough lattices R (X), R (Y) ⊆U we have

i. Io$\underline{R}$ (X)⊂ $X \subseteq \overline{R}$ (X)
ii. $\underline{R}$ (X∩ Y) = $\underline{R}$ (X) ∩ $\underline{R}$ (Y)
iii. $\overline{R}$ (X∪ Y) = $\overline{R}$ (X) ∪ $\overline{R}$ (Y)
iv. X⊆Y ⇒ $\underline{R}$ (X) ⊆ $\underline{R}$ (Y) &$\overline{R}$ (X)⊆ $\overline{R}$ (Y)
v. $\underline{R}$ (X∪ Y)⊇ $\underline{R}$ (X) ∪ $\underline{R}$ (Y)
vi. $\overline{R}$ (X∩ Y)⊆ $\overline{R}$ (X) ∩ $\overline{R}$ (Y)
vii. $\underline{R}$ ($\underline{R}$ (X)) = $\underline{R}$ (X)
viii. $\overline{R}$ ($\overline{R}$ (X)) = $\overline{R}$ (X).

3.3 Theorem:

Let (U, E) be an approximation space. Let E be an equivalence relation on U. Let L be a complete lattices with n≥3 elements then

(i) Number of definable sub lattice of L under the relation E is equal to 2^m where m is the number of equivalence classes

(ii) Number of undefinable (rough) sub lattice of L under the relation E is equal to 2^n - 2^m where n is the number of elements of L**.**

3.4 Theorem:

Let R (X)& R (Y) be two rough sub lattices of a rough lattice R (L). A sufficient condition for intersection of two rough sub lattices of a rough lattice to be a rough sub lattice is $\overline{R}\,(X) \cap \overline{R}\,(Y) = \overline{R\,(X) \cap R\,(Y)}$

Proof:

Suppose R (X)& R (Y) be two rough sub lattices of a rough lattice R (L). It is obvious that R (X) ∩ R (Y) ⊆R (L). Consider a, b ∈ R (X) ∩ R (Y). Since R (X)& R (Y) are rough sub lattices. We have a∧b, a∨b∈ $\overline{R}\,(X)$ & a∧b, a∨b∈ $\overline{R}\,(Y)$, that is a∧b∈ $\overline{R}\,(X) \cap \overline{R}\,(Y)$ & a∨b ∈ $\overline{R}\,(X) \cap \overline{R}\,(Y)$ Since $\overline{R}\,(X) \cap \overline{R}\,(Y) = \overline{R\,(X) \cap R\,(Y)}$ we have a∧b∈$\overline{R\,(X) \cap R\,(Y)}$ & a∨b ∈ $\overline{R\,(X) \cap R\,(Y)}$

Hence R (X) ∩ R (Y) is a rough sub lattice of R (L).

3.5 Example:

Let U = $\{0, a, b, c, d, 1\}$. We define the binary relation ≤on U. Let the equivalence classes are $[0]_E = \{0, a\}$, $[b]_E = \{b\}$, $[c]_E = \{c\}$, $[1]_E = \{1, d\}$ Take $X_1 = \{a, b, d\}$ $\overline{R\,(X_1)} = \{0, a, b, , d, 1\}$, X_1 is a rough lattice. Let $X_2 = \{a, b, c\}$ $\overline{R\,(X_2)}$ = {0, a, b, c}here $b, c \in X_2$ but b∨c ∉ $\overline{R\,(X_2)}$ therefore X_2 is not a rough lattice.

Let $X_3 = \{a, b\}$ $\overline{R(X_3)} = \{0, a, b\}$ Thus we have X_3 is a rough sub lattice of a rough lattice X_1. The membership values of X_1 in U are $\mu_{X_1}(a) = \frac{|[a]_R \cap X_1|}{|[a]_R|} = 1, \mu_{X_1}(b) = \mu_{X_1}(d) = 1, \mu_{X_1}(0) = \mu_{X_1}(1) = \frac{1}{2}, \mu_{X_1}(c) = 0$

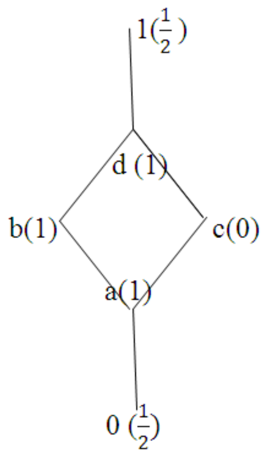

Figure 1

Weighted Rough Lattice

3.6 Theorem:
Let (U, E)be an approximation space. Let E be a congruence relation on U. If X ⊆U, then (i)If X is join semi lattice in U, then $\overline{R}(X)$ also join semi lattice in U (ii)If X is meet semi lattice in U, then $\overline{R}(X)$ also meet semi lattice in U.

3.7 Theorem:
Let (U, E)be an approximation space. Let E be a congruence relation on U. If X ⊆U, X is upset& meet semi lattice in U, then upper approximation $\overline{R}(X)$ also upset& meet semi lattice in U.

3.8 Theorem:
Let (U, E)be an approximation space. Let E be a congruence relation on U. If X, Y ⊆U, X&Y are down sets & join semi lattice in U, then the following are hold.

(i) $\overline{R}(X \wedge Y) = \overline{R}(X) \wedge \overline{R}(Y$

(ii) $\overline{R}(X) \vee \overline{R}(Y) \subseteq \overline{R}(X \vee Y)$

(iii) $\overline{R}[\overline{R}(X) \vee \overline{R}(Y)] = \overline{R}(X \vee Y)$

3.9 Theorem:
Let (U, E) be an approximation space. Let E be a congruence relation on U.
If X ⊆U, then

(1) If X is down set in U, then (i) $\overline{R}\ (X)\ =\ D\ (\overline{R}\ (X))_U$ (ii) $\underline{R}$ (X) $=\ D\ (\underline{R}\ (X))_U$

(2) If X is upset in U, then (i) $\overline{R}\ (X)\ =\ U\ (\overline{R}\ (X))_U$ (ii) $\underline{R}$ (X) $=\ U\ (\underline{R}\ (X))_U$

3.10 Corollary:

Let (U, E) be an approximation space. Let E be a congruence relation on U.
If X ⊆U, then We have

(i) $D\ (\overline{R}\ (X))_U\ =\ \overline{R}(D(X)_U)\ =\ D\ (\overline{R}(D(X)))_U$

(ii) $U\ (\overline{R}\ (X))_U\ =\ \overline{R}(U(X)_U)\ =\ U\ (\overline{R}(U(X)))_U$

(iii) $\overline{R}\ [X \vee \overline{R}\ (Y)] = \overline{R}$ (X$\vee$ Y).

3.11 Corollary:

Let (U, E) be an approximation space. Let R be a congruence relation on U. If X ⊆U, X is a down set& join semi lattice in U, for every a, b ∈ X then

(i) $\overline{R}\ (D\ (a \wedge b)_U) =\ \overline{R}\ (D\ (a)_U) \wedge\ \overline{R}\ (D\ (b)_U)$

(ii) $\overline{R}\ (D\ (a \vee b)_U) =\ \overline{R}\ (D\ (a)_U) \vee\ \overline{R}\ (D\ (b)_U)$

3.12 Theorem:

Let (U, E) be an approximation space. Let E be a congruence relation on U. then

(i) the set of all down set of a Rough poset are Rough poset.

(ii) The set of all up set of a Rough poset are Rough poset

3.13 Theorem:

Let (U, E) be an approximation space. Let E be a congruence relation on U. then

(i) The set of all Rough set of a lattice need not be a rough lattice.

(ii) The set of all down set of a lattice need not be a rough lattice

3.14 Homomorphism & Isomorphism of rough lattice:

Let (U_1, E_1) & (U_2, E_2)be two approximation spaces and $\vee$,$\wedge$&$\overline{\vee}$, $\overline{\wedge}$ be operations over universes U_1 & U_2 respectively. Let R (L_1)⊂U_1, R (L_2) ⊂U_2, R (L_1), R (L_2)are called rough homomorphism lattices if there exist a surjection φ : R (L_1) → R (L_2) such that

(i) Φ $(x \vee\ y)$ = φ $(x)\overline{\vee}$ φ (y)

(ii) Φ $(x \wedge\ y)$ = φ $(x)\overline{\wedge}$ φ (y) $\forall\ x, y$∈R (L_1)

3.15 Theorem:

Let R (L_1)⊂U_1, R (L_2) ⊂U_2, R (L_1), R (L_2)are called rough homomorphism lattices. If $\vee$,$\wedge$ satisfies commutative law then$\overline{\vee}$, $\overline{\wedge}$also satisfies it.

3.16 Theorem:

Let R (L_1)⊂U_1, R (L_2) ⊂U_2, R (L_1), R (L_2)are called rough homomorphism lattices. If φ ($\overline{R}$ (L_1)) = $\overline{R}$ (L_2), then φ (R (L_1)) is a rough lattice.

3.17 Complete rough lattice:
A complete rough lattice is a rough lattice (R (L), $\leq$) such that any sub set X $\subseteq$R (L) has a lub and glb in R (L).

3.18 Bounded rough lattice:
A bounded rough lattice is a rough lattice (R (L), $\leq$) which has an upper bound and lower bound

3.19 Theorem:
A complete rough lattice is a bounded rough lattice but the converse need not be true.

3.20 Example:
Let R = (N, E) be an approximation space where N- natural numbers. Let E be an equivalence relation. We define the binary relation $\leq$on U and the operator's are $\wedge$-lower bound, $\vee$ -upper bound. Let X$\subseteq$Nand let X = {finite even integers} and $\overline{R}\ (X)$ = {finite even integers}. It has upper bound and lower bound.
Therefore X = R (L) = {Y$\subseteq$N: Y is finite and contains only even integers} is a rough lattice as well as bounded rough lattice but it is not a complete rough lattice. Since the set R (L) = {X$\subseteq$N: X is finite and contains only even integers} has no least upper bound.

3.21 Rough Boolean Algebra:
A complemented distributive rough lattice is called a rough Boolean algebra

3.22 Theorem:
Let (U, E)be an approximation space. Let $\underline{R} : P\ (U) \rightarrow P\ (U)$& $\overline{R} : P\ (U) \rightarrow P\ (U)$ be the approximation operators then rough set algebra (P (U), ^{c}, $\underline{R}, \overline{R}$,$\cup$,$\cap$) need not be a rough Boolean algebra.

3.23 Example:
Let U = {a, b, c, d}. Let the equivalence classes are $[a]_R = [b]_R$ = {a, b}, $[c]_R$ = {c}, $[d]_R$ = {d}. Let X $\subseteq$U then

Table 1

X	$\underline{R}$ **(X)**	$\overline{R}\ (X)$	μ **(X)**
{a}	ϕ	{a, b}	0
{b}	ϕ	{a, b}	0
{c}	{c}	{c}	1
{d}	{d}	{d}	1
{a, b}	{a, b}	{a, b}	1
{a, c}	{c}	{a, b, c}	1/3
$\{a, d\}$	$\{d\}$	$\{a, b, d\}$	1/3
{b, c}	{c}	{a, b, c}	1/3

{b, d}	{d}	{a, b, d}	1/3
{c, d}	{c, d}	{c, d}	1
{a, b, c}	{a, b, c}	{a, b, c}	1
{a, b, d}	{a, b, d}	{a, b, d}	1
1/2{a, c, d}	{c, d}	{a, b, c, d}	1/2
{b, c, d}	{c, d}	{a, b, c, d}	1/2
{a, b, c, d}	{a, b, c, d}	{a, b, c, d}	1/2
ϕ	ϕ	ϕ	1

The number of rough subset of U = $12 \neq 2^4$. Since every Boolean algebra is isomorphic to a power set. Hence every Boolean algebra contains 2^U elements. Therefore rough set algebra is not a Boolean algebra.

4. Information Systems

In the Rough Set terminology, a data set is represented as a table, where each row represents a case, an event, a patient, or simply an object. Every column represents an attribute (a variable, an observation, a property etc) that can be measured for each object; the attribute may be also supplied by a human expert or user. This table is called an *information system*. More formally, it is a pair $\mathcal{A}$ = (U, A) where U is a non-empty finite set of *objects* called the *universe* and A is a non-empty finite set of *attributes* in such a way that $a:U \to V_a$ for every $a \in A$. The set V_a is called the *value set* of a. The main object of RS data analysis is to reduce data size.

4.1 Example

Consider the simple information system with seven objects (cases), with four condition attributes{a_1, a_2, a_3, a_4}and decision attribute{d}

Table 2

attributes \ objects	a_1	a_2	a_3	a_4	$\boldsymbol{d}$
x_1	2	2	2	4	**1**
x_2	1	1	1	3	**2**
x_3	2	2	2	4	**1**
x_4	3	3	1	1	**3**
x_5	4	2	1	2	**4**
x_6	4	2	1	3	**2**
x_7	3	2	2	1	**3**
x_8	4	3	2	3	**2**

4.2 Reduct:

A reduct is a reduction of an information system which results in no loss of information by **removing conditional attributes**. There may be one or many for a given information system. Let (U, A, R) be an approximation space, which U = $\{x_1, x_2, \ldots, x_n\}$ is the set of objects, each xi (i≤n) called an object; A = $\{a_1, a_2, \ldots a_m\}$ is the set of attributes, each aj (j≤m) called an attribute, R is binary. For any app space (U, A, R), there must be reduction sets

If for any a∈ A, (U, A-{a}, $R_{A-\{a\}}$) ≠ (U, A, R) then A is a reduction set.

If for any a∈ A, (U, A-{a}, $R_{A-\{a\}}$)≅ (U, A, R) then A_1 = A-{a}is a reduction set.

If for any a∈ A_1, (U, A_1-{a}, $R_{A-\{a\}}$) ≠ (U, A, R) then A_1 is a reduction set. Otherwise study $A_2 = A_1$ –{a} repeat the above process, we obtain the reduction sets.

4.3 Core attribute:

A core is the set of attributes which are common to all reducts. The set of attribute A can be divided into the following three categories.

1. The set of core attributes = ∩ D_i (D -reduction sets)
2. The set of necessary attributes = ∪ D_i - ∩ D_i
3. The set of unnecessary attributes = A- ∪ D_i

4.4 Relation :

Relation between each object is defined by

$d\ (x_i, x_j) = \sum_{k=1}^{m}\left(a_{ik} - a_{jk}\right)^2$

4.5 Definition :

Let (U, A, R) be an approximation space, which U = $\{x_1, x_2, \ldots, x_n\}$ is the set of objects, each $xi\ (i \le n)$ called an object; A = $\{a_1, a_2, \ldots a_m\}$ is the set of attributes A **rough data set lattice (RDS Lattice)**is denoted by $(RU, \le), RU = (\phi \cup U \cup RU_1 \cup \ldots \cup RU_n)$

Where $RU_1 = \{x_1, x_j \,/\, d\ (x_1, x_j) \le r, x_1, x_j \in U, j = 1 \ldots n,\}$

…

$RU_n = \{x_n, x_j \,/\, d\ (x_n, x_j) \le r, x_n, x_j \in U, j = 1 \ldots n,\}$

4.6 Definition:

Let (U, A, R) be an approximation space, which U = $\{x_1, x_2, \ldots, x_n\}$ is the set of objects, each $xi\ (i \le n)$ called an object; A = $\{a_1, a_2, \ldots a_m\}$ is the set of attributes A **weighted rough data set lattice (WRDS Lattice)** is denoted by $((RU, RA) \le)$ such that $RU = (\phi \cup U \cup RU_1 \cup \ldots \cup RU_n)$

$RA = (\phi \cup RA_0 \cup RA_1 \cup \ldots \cup RA_n)$

Where

$RU_1 = \{x_1, x_j \,/\, d\ (x_1, x_j) \ge r, x_1, x_j \in U, j = 1 \ldots n,\}$

…

$RU_n = \{x_n, x_j \,/\, d\ (x_n, x_j) \ge r, x_n, x_j \in U, j = 1 \ldots n,\}$,

$RA_1 = \{\frac{\sum a_{k1}}{|RU_1|}, \frac{\sum a_{k2}}{|RU_1|}, \ldots \frac{\sum a_{km}}{|RU_1|} / there\ exist\ some\ x_k \in RU_1\}$

…

$$RA_n = \{\frac{\sum a_{k1}}{|RU_n|}, \frac{\sum a_{k2}}{|RU_n|}, \dots \frac{\sum a_{km}}{|RU_n|} / there\ exist\ some\ x_k \in RU_1\}$$

$$\&RA_0 = \{\frac{\sum a_{k1}}{|U|}, \frac{\sum a_{k2}}{|U|}, \dots \frac{\sum a_{km}}{|U|} \frac{}{for}\ all\ x_k \in U\}$$

5. Find rough data set lattice (RDS lattice)

To compute RU for finding the rough data set lattice of the given information system consists of the following steps.

A. Find the relation for each object.
B. Choose relational parameter r.
C. Compute $RU_1, RU_{2.} \dots \& RU_n$`

This paper using relational parameter $r = 4$. We can obtain initial rough data set lattice using the above three steps. RU = {ϕ, U, $\{x_1, x_2, x_3\}$, $\{x_4, x_5, x_7\}$, $\{x_5, x_4, x_6, x_8\}$, $\{x_7, x_4\}$, $\{x_8, x_5, x_6,\}$

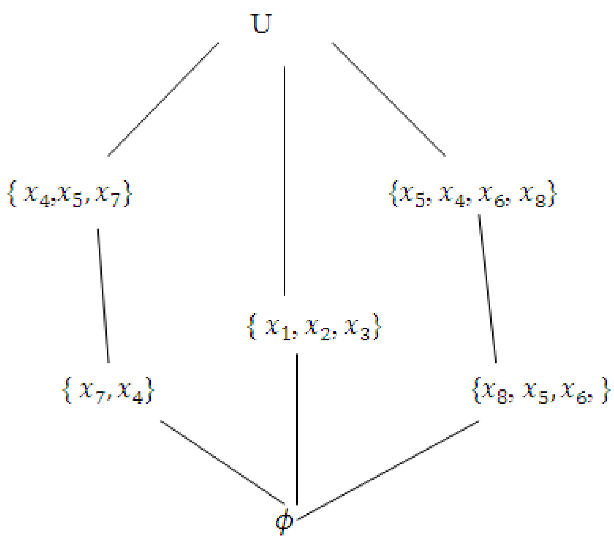

Figure 2: RDS lattice

6. Find the core in rough data set lattice

Finding core in rough data set lattice of the given information system consists of the following three steps.

a) Calculate attribute reduction.
b) Find all reduct sets.
c) Find core attribute.
d) Compute RU

6.1 a)Calculate attribute reduction

$R_{a_1} = \sum_{k=1}^{7} a_{k1} = 21$, $R_{a_2} = \sum_{k=1}^{7} a_{k2} = 17$, $R_{a_3} = \sum_{k=1}^{7} a_{k3} = 12$, $R_{a_4} = \sum_{k=1}^{7} a_{k3} = 22$

$R_{a_3} < R_{a_2} < R_{a_1} < R_{a_4}$

Priority given to the attribute a_3. First we delete the attribute a_3 and then delete a_2 etc

6.2 b)Find reduct sets

(i)Find D_1: Finding **reduct set** of the given rough data set consists of the following three steps

1. Deleting the attribute a_3
2. Find the compatible relation for each object.
3. Compute RU.

We can obtain rough data set lattice using the above three steps.

RU = $\{\phi, \{x_1, x_2, x_3, x_4, x_5\ x_6, x_7, x_8\}, \{x_1, x_2, x_3\}, \{x_4, x_5, x_7\}, \{x_5, x_4, x_6, x_8\}, \{x_7, x_4\}, \{x_8, x_5, x_6,\}\}$

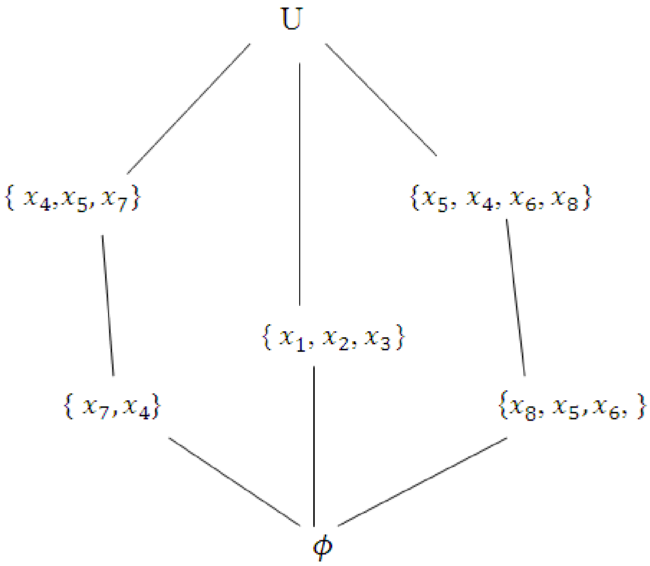

Figure 3: RDS lattice (A-a_3)

By definition, the reduction set $D_1 = \{a_1, a_2, a_4\}$

(ii)Find D_2: we delete attribute a_2. We obtain the same rough data set lattice. RU = $\{\phi, \{x_1, x_2, x_3, x_4, x_5\ x_6, x_7, x_8\}, \{x_1, x_2, x_3\}, \{x_4, x_5, x_7\}, \{x_5, x_4, x_6, x_8\}, \{x_7, x_4\}, \{x_8, x_5, x_6,\}\}$

Now the reduction set $D_2 = \{a_1, a_3, a_4\}$

(iii)Find D_3 :we delete attribute a_1. We obtain the different rough data set lattice RU = $\{\phi, U, \{x_1, x_2, x_3, x_6, x_8\}, \{x_2, x_1, x_3\ x_5, x_6\}, \{x_3\ x_1, x_2, x_5\ x_6, x_8\}, \{x_4, x_2, x_5, x_7\} \{x_5\ x_2, x_4, x_6\ x_7, x_8\}, \{x_6\ x_1, x_2, x_3\ x_5, x_8\}, \{x_7, x_4, x_5\}, \{x_8\ x_1, x_3, x_5\ x_6\}$

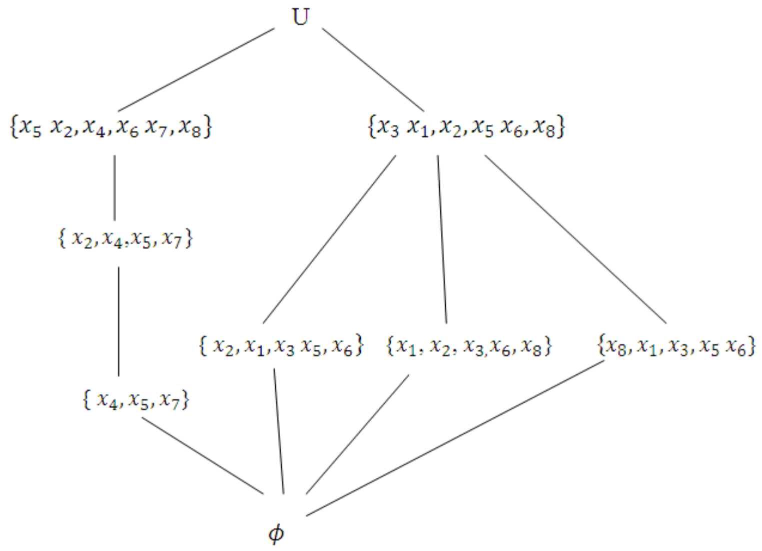

Figure 4: RDS lattice (A-a_1)

Now the reduction set $D_3 = \{a_1, a_2, a_3, a_4\}$

(iv) Find D_4 : We delete attribute a_3. We obtain different rough data set lattice. Therefore the reduction set $D_4 = \{a_1, a_2, a_3, a_4\}$

6.3 c)Find core

The reduction sets are $D_1 = \{a_1, a_2, a_4\}$, $D_2 = \{a_1, a_3, a_4\}$, $D_3 = \{a_1, a_2, a_3, a_4\}$ $D_4 = \{a_1, a_2, a_3, a_4\}$ By the definition of core, the core attribute set = $\{a_1, a_4\}$. we delete all the redundancy attributes a_2&a_3, there are two core attributes a_1&a4, Table 2 can be reduced to table3 as follows

Table 3

attributes / objects	a_1	a_4	d
x_1	2	4	**1**
x_2	1	3	**2**
x_3	2	4	**1**
x_4	3	1	**3**

x_5	4	2	**4**
x_6	4	3	**2**
x_7	3	1	**3**
x_8	4	3	**2**

IND (U/$\{a_1, a_{4,}$d}) = $\{\{x_1, x_3\}, \{x_2\}, \{x_{4,}x_7\}, \{x_5\}, \{x_6, x_8\}\}$. Reduce table 3 by eliminating the same values of decision and condition attributes, that is we can merge different rows that has the same values for decision and condition attributes. This method is called object reduction.

Table4

attributes / objects	a_1	a_4	$\boldsymbol{d}$
$\{x_1, x_3\} = y_1$	2	4	**1**
$\{x_2\} = y_2$	1	3	**2**
$\{x_4, x_7\} = y_3$	3	1	**3**
$\{x_5\} = y_4$	4	2	**4**
$\{x_6, x_8\} = y_5$	4	3	**2**

6.4 d) Compute RU

Using table 4 we obtain the core rough data set lattice.

RU = $\{\{\phi, \{y_1, y_2, y_3, y_{4,}y_5\}\{y_1, y_2\}, \{y_3, y_4\}, \{y_4, y_5\}, \{y_3, y_4, y_5\}\}$

Table 3 shows the core of the given information table. Using table 4 we obtain the core weighted rough data set lattice.

7. Find the weighted rough data set

Finding the weighted rough data set lattice consists of the following steps using table4.

1. Compute RU
2. Compute RA

RU = $\{\{\phi, \{y_1, y_2, y_3, y_{4,}y_5\}\{y_1, y_2\}, \{y_3, y_4\}, \{y_4, y_5\}, \{y_3, y_4, y_5\}\}$

$RA_1 = \{\frac{3}{2}, \frac{7}{2}\}$, $RA_2 = \{\frac{3}{2}, \frac{7}{2}\}$, $RA_3 = \{\frac{7}{2}, \frac{3}{2}\}$, $RA_4 = \{\frac{11}{3}, \frac{6}{3}\}$, $RA_5 = \{\frac{8}{2}, \frac{5}{2}\}$

RA = $\{\phi, \{\frac{3}{2}, \frac{7}{2}\}, \{\frac{7}{2}, \frac{3}{2}\}, \{\frac{11}{3}, \frac{6}{3}\}, \{\frac{8}{2}, \frac{5}{2}\}\}$

(RU, RA) = $\{\{\phi, \phi\}$ $(U, \{\frac{14}{5}, \frac{7}{2}\})$, $(\{y_1, y_2\}\{\frac{3}{2}, \frac{7}{2}\})$, $(\{y_3, y_4\}\{\frac{7}{2}, \frac{3}{2}\})$, $(\{y_4, y_5\}\{\frac{11}{3}, \frac{6}{3}\})$, $(\{y_3, y_4, y_5\}\{\frac{8}{2}, \frac{5}{2}\})\}$

We obtain the core weighted rough data set lattice

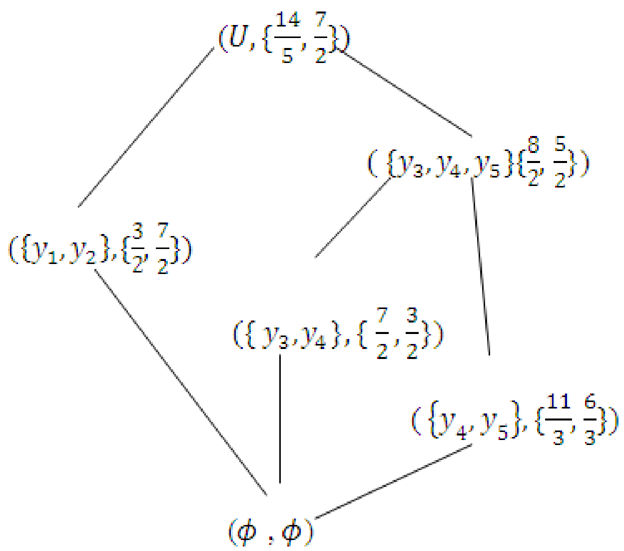

Figure 5: Core WRDS lattice

8. Conclusion

a_2 and a_3 are redundancy attribute, attributes a_1&a_4 are the core attributes. The fig 5 is the core lattice of the given rough data set lattice. The structure of the rough data set lattice of fig 5 is more stable.

We make a further study to the combination of lattice theory and graph theory. lattice theory posses a sophisticated mathematical structure. And it has been widely used in real world. So we hope our work in this paper could be contributive to the theoretical development and applications of rough set.

References

[1]. A. A. Estaji, M. R. Hooshmandasal, B. Davvaz " Rough set theory applied to lattice theory"Elseveir, Information sciences 200 (2012)108-122.

[2]. R. Wille "Restructuring lattice theory: An approach based on hierarchies of concept", Rival I, ed. ordered sets, (1982)

[3]. L. Lan "Attribute reduction based on information entropy and its application" Liao Ning:Dalian Maritime university, (2008)

[4]. Y. Jianfeng "Multi value concept lattice and intervalue concept lattice structure and attribute reduction" Northwestern university (2012).

[5]. Hongcan Yan Feng Zhang, Baoxiang Liu "Multi value- Attribute concept lattice reduction Based on Granular computing" International journal of Grid and distributed computing Vol. 9, No 1 (2016)pp79-88.

[6]. Z.. Pawlak, Rough sets. :International journal of computer and information sciences, 11, 341-356, 1982.

[7]. Z.. Pawlak, Rough sets, Theoritical aspects of reasoning about data, Kluwer Academic Publishers, Dordrecht, 1991.

[8]. Y. Y. Yao :and Y. Chen, Rough set approximation in formal concept analysis, manuscript, 2004.

[9]. B. A. Davey& H. A. Priestley, 2002, ”Introduction to lattice and order” Cambridge university press, 2nd edition, 298P.

[10]. G. Birkhoff, 1979”lattice theory” American mathematical society, Calloquium publications, vol25, 3rd edition, 418p.

[11] G. Gratzer 1998 “General lattice theory” Birkhuser verlag, Basel, 2nd edition, 663p.

[12] Anjan Mukherjee, S. Bhattacharya Halder “Fuzzy sets and Fuzzy topology” –, 2015.